212099

M.Sc.(Semester-I) Examination, December 2021

MATHEMATICS

Paper - I

(Advanced Abstract Algebra-I)

Time Allowed : 3 hours — *Maximum Marks Regular/Private : 40/50*

नोट : स्वाध्यायी परीक्षार्थियों के लिए पूर्णांक 50 अंक होंगे, खण्ड 'अ'- 5, खण्ड 'ब'-15 एवं खण्ड 'स'-30 अंकों का होगा।

Note : All questions are compulsory.

Section-A

(Objective Type Questions) **5×1=5**

1. Choose the correct answer :

(i) If G be a group, z its centre and G/z is cyclic, Then G must be :

(a) Abelian (b) Non-Abelian

(c) Normalizer (d) None of these

(ii) Jordan-Hölder theorem is a theorem about composition series of :

(a) Abelian group (b) Finite group

(c) Infinite group (d) None of these

P.T.O.

(iii) If G is solvable, and H is a subgroup of G, then H is :

(a) Nilpotent group (b) Solvable

(c) Super solvable (d) None of these

(iv) Every finite extension of a field is :

(a) Finite extension

(b) Transcendental extension

(c) Algebraic extension

(d) None of these

(v) Any two finite fields having the same number of elements are :

(a) Isomorphic (b) Equivalent

(c) Homomorphic (d) None of these

Section-B

(Short Answer Type Questions) 5×2=10

Note : Marks : 10 (5 questions of 2 marks each)

2. Define Normalizer of an element.

OR

Define self conjugate elements.

3. Prove that every finite group has composition series.

OR

Write the statement of Schreier's Refinement theorem.

4. Define commutator subgroup of a group.

OR

Prove that every homomorphic image of solvable group is solvable.

5. Define Algebraic element and give an example.

OR

Define Transcendental extension and give an example.

6. Define separable extension.

OR

Define perfect field.

Section-C

(Long Answer Type Questions) 5×5=25

Note : Marks : 25 (5 questions of 5 marks each)

7. State and prove cauchy's theorem for non abelian group.

OR

Let G be a finite group and let P be a prime. If $P^n \mid o(G)$ but $P^{n+1} \nmid o(G)$, then any two subgroups of G of order P^n are conjugate.

8. State and prove ZassenLaus Lemma.

OR

State and prove Jordan Holder's theorem.

9. Prove that a group G is solvable if and only if $G^{(k)} = (e)$ for some integer K.

OR

Prove that every Nilpotent group is solvable.

10. Let $F \subseteq E \subseteq K$ be fields. If K is a finite extension of E and E is a finite extension of F, then K is a finite extension of F and $[K:F]=[K:E][E:F]$.

OR

Define splitting field with examples and prove that splitting fields are algebraic extensions.

11. Prove that every finite separable extension of a field is necessarily a simple extension.

OR

Prove that the multiplicative group of non-zero elements of a finite field is cyclic.

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