212103

M.Sc. (Semester-I) Examination, December 2021

MATHEMATICS

Paper - V (Optional -I)

(Advanced Discrete Mathematics-I)

Time Allowed : 3 hours *Maximum Marks : 40*

Regular/Private : 40/50

नोट : स्वाध्यायी परीक्षार्थियों के लिए पूर्णांक 50 अंक होंगे, खण्ड 'अ'- 5, खण्ड 'ब'-15 एवं खण्ड 'स'-30 अंकों का होगा।

Note : (1) All sections are compulsory. Marks are indicated against each section.

(2) Symbols have their useual meanings.

Section-A

(Objective Type Questions) 5×1=5

1. Choose the correct option :

(i) A subgroup <H, *> of <G, *> is called normal subgroup if :

(a) $Ha \leq aH$ (b) $Ha = aH$

(c) $Ha \geq aH$ (d) $Ha \neq aH$

(ii) Let $(P,\leq)$ be a partially ordered set, an element $m \in p$ is said to be a maximal element if :

(a) $m \leq x \Rightarrow m = x$ (b) $m \geq x \Rightarrow m = x$

(c) $m \leq x \Rightarrow m \neq x$ (d) $m \geq x \Rightarrow m \neq x$

(iii) A Boolean algebra can not have :

(a) Two elements (b) Four elements

(c) Three elements (d) Five elements

(iv) A vertex of degree 1 is :

(a) A pendant vertex

(b) An isolated vertex

(c) A vertex having loop also on it

(d) None of these

(v) Every cut-set in connected graph G contains atleast of every spanning tree of G :

(a) One (b) Two

(c) Three (d) Four

Section-B

(Short Answer Type Questions) 5×2=10

2. Define semigroup with example.

OR

Define Homomorphism of Semigroup and monoid.

3. Define sublattice with example.

OR

Define the following –

(i) Bounded lattice (ii) Isomorphic lattice

4. Define Boolean algebra with example.

OR

Define the following :

(i) Boolean Function (ii) Minimal Boolean function

5. Define the following :

(i) Degree of a vertex (ii) Isolated vertex

(iii) Pendant vertex

OR

Define Tree with example.

6. Define Biportite graph with example.

OR

Define Spanning Tree with example.

Section-C

(Long Answer Type Questions) 5×5=25

7. Let $< M,*,e >$ and $<T,\Delta,e')$ be two monoids with identities e and e' if f is an onto mapping from M to T i.e.

$f : M \to T$ is an isomorphism then prove that $f(e) = e'$

OR

Let g be a homomorphism of $< G,* >$ on to $< G',\Delta >$ with kernal K. Then $<G/K,.>$ is isomorphic to $<G',\Delta>$.

8. Show that the dual of a lattice is a lattice.

OR

Show that every chain is a distributive lattice.

9. If a and b are arbitrary elements of a Boolean algebra B then show that :

(i) $(a+b)' = a'b'$ (ii) $(ab)' = a'+b'$

OR

Define the following :

(i) Switching circuit (ii) Parallel circuit

(iii) Series circuit

10. The sum of the degree of all vertices in a graph is equal to twice the number of edges.

OR

The maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$

11. Prove that Every connected graph has at least one spanning tree.

OR

State the Kruskal's algorithm. Find the minimal spanning tree for the following graph –