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212102

M.Sc. (Semester-I) Examination, December 2021

MATHEMATICS

Paper - IV

(Complex Analysis-I)

Time Allowed : 3 hours

Maximum Marks
Regular/Private : 40/50

नोट : स्वाध्यायी परीक्षार्थियों के लिए पूर्णांक 50 अंक होंगे, खण्ड 'अ'– 5, खण्ड 'ब'–15 एवं खण्ड 'स'–30 अंकों का होगा।

Section-A

(Objective Type Questions) 5×1=5

Note : Attempt all questions. Each question carries one mark.

1. Choose the correct option.

(i) The value of $\int_C \frac{dz}{z-\alpha}$ where C is the circle with centre at α and radius r is –

(a) $3\pi i$ (b) $2\pi i$

(c) πi (d) $4\pi i$

P.T.O.

(ii) Which theorem is a sort of converse of Morera's theorem -

(a) Taylor's theorem

(b) Liouville's theorem

(c) Cauchy's theorem

(d) None of these

(iii) If a function $f(z)$ has a pole of order m at $z=a$, then the function ϕ defined by $(z) = (z-a)^m f(z)$ and $\phi(a) \neq 0$ has -

(a) a removable singularity at a

(b) a isolated singularity at $z = a$

(c) a pole

(d) None of these

(iv) The residue of $\frac{1}{z^2+a^2}$ at $z=ai$ is -

(a) $-\frac{1}{2ai}$ (b) $\frac{1}{2a^2}$

(c) $-\frac{1}{2a^2}$ (d) $\frac{1}{2ai}$

(v) The fixed points of the bilinear transformation $w = \frac{z-1}{z+1}$ is -

(a) i and $-i$ (b) 1 and -1

(b) 1 and i (d) -1 and i

Section-B

(Short Answer Type Questions) 5×2=10

Note : Attempt all five questions. Each question carries two marks.

2. Evaluate by Cauchy's integral formula intergral $\int_c \frac{dz}{z(z+\pi i)}$ where c is circle $|z+3i|=1$.

OR

Find the value of integral $\int_0^{1+i} z^2 dz$ along the line segment from $z=0$ to $z=1+i$.

3. If C is a closed contour containing the origin inside it prove that :

$$\frac{a^n}{n!} = \frac{1}{2\pi i}\int_c \frac{e^{az}}{z^{n+1}} dz$$

OR

Expand log $(1+z)$ in a Taylor's series about $z = 0$ and determine the region of convergence for the resulting series.

4. State and prove argument principle.

OR

Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $|z| < 1$.

5. Find the residue of $\dfrac{1}{(z^2+a^2)^2}$ at $z = ai$.

OR

State and prove Cauchy's residue theorem.

6. Define Bilinear transformation and inverse Bilinear transformation.

OR

Prove that the cross-ratio are invariant under a bilinear transformation.

Section-C

(Long Answer Type Questions) **5×5=25**

Note : Attempt all questions. Each question carries five marks.

7. State and prove Cauchy's integral formula.

OR

If the function $f(z)$ is analytic and single-valued inside and on a simple closed contour C, then show that

$$\int_c f(z)dz = 0$$

8. State and prove Morera's theorem.

OR

State and prove Liouville's theorem.

9. State and prove Rouche's theorem.

OR

State and prove Schwarz lemma.

10. Evaluate the residue of $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at the poles $z = 1, 2, 3$.

OR

Show that

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}} \text{ where } a > b > o$$

11. Prove that the set of all bilinear transformations forms a non-abelian group under the product of transformations.

OR

Find the bilinear transformation which maps $z = 1, i, -1$ respectively onto $w = i, 0, -i$.

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